Eddy Viscosity for Variable Density Coflowing Streams

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Nomenclature

b	= width of mixing region in Prandtl model
K_F	= constant in Ferri, Libby, Zakkay model
K_p	= empirical constant in Prandtl model
$K(\lambda)$	= potential core "constant" dependent on mass flux ratio
$r_{1/2}$	= width of mixing region in Ferri, Libby, Zakkay model
U	= jet exit velocity
u	= streamwise velocity
u_{max}, u_{min}	= maximum and minimum velocities in Prandtl model
ϵ	= eddy viscosity
$\lambda = \rho u_e/\rho_o U$	= mass flux ratio
ξ	= concentration by volume
ρ	= density
Subscripts	
c	= centerline value
e	= external coflowing stream value
o	= initial value

Theme

EDDY viscosity models have proven to be a very popular technique in the mathematical treatment of turbulent flows. Their popularity stems in large part from their simplicity as well as the fact that their introduction allows the use of laminar methods of solution for turbulent problems.

The purpose of the present paper is to modify the classical Prandtl eddy viscosity model (which was formulated for an incompressible jet into a still air) to account for a density difference between the jet and a nonzero velocity coflowing stream. A simultaneous objective is to retain the relative computational simplicity of the model.

The primary motivation for this study is to develop a simple tool to calculate the behavior of variable density jets for application to thrust augmenting ejector problems. Thus the coflowing stream is due solely to entrainment by the jet, and the mass flux ratio, $\lambda = \rho_e u_e/\rho_o U$, is always less than unity.

Contents

The classical eddy viscosity model proposed by Prandtl[1] for the calculation of freejet problems is

$$\epsilon = K_p b(u_{max} - u_{min}) \quad (1)$$

The constant K_p has been chosen[2] to be equal to 0.014 for $u_c \geq 0.99U$ and equal to 0.022 for $u_c < 0.99U$.

The predictions of the Prandtl model are reasonably good[2] in the case of an incompressible freejet exhausting into still air, the case for which the model was formulated. However, the predictions of the model suffer with the addition of a coflowing stream and/or a nonuniform density. Several models based on the Prandtl model have been proposed to alleviate these problems.

In order to include the effect of variable density through the jet flow, Ferri, Libby, and Zakkay[3] proposed a model similar to the Prandtl model, but dependent upon the difference in the mass fluxes rather than the difference in velocity. Thus

$$\rho\epsilon = K_F r_{1/2}(\rho_c u_c - \rho_e u_e) \quad (2)$$

where $r_{1/2}$ is defined at any streamwise station as the distance between the centerline and the position at which the mass flux is equal to the average of the centerline value $\rho_c u_c$ and the coflowing stream value, $\rho_e u_e$. In addition, the constant K_F was chosen to be equal to 0.025. The Ferri model does improve the predictions of some variable density flows, but generally underestimates their decay rates.

A model designed to include the effect of a coflowing stream on constant density jet mixing was proposed by Viets.[4] That this model is also based upon the Prandtl model may be seen from the expression

$$\epsilon = K_p b(u_c - u_e)\left[\frac{U - u_e}{u_c - u_e}\right]^{u_e/U} \quad (3)$$

The bracketed term raised to the power of the initial velocity ratio is the only difference between Eq. (3) and the Prandtl model, Eq. (1). This term arises from an argument concerning the size of the lumps of fluid in the Prandtl model. The modified Prandtl model leads to improved prediction of the decay of constant density coaxial jets in coflowing streams.

If Eq. (3) is rewritten in terms of mass flow instead of velocity, the result is

$$\rho\epsilon = K(\lambda)b(\rho_c u_c - \rho_e u_e)\left[\frac{\rho_o U - \rho_e u_e}{\rho_c u_c - \rho_e u_e}\right]^{\lambda} \quad (4)$$

where $\lambda = \rho_e u_e/\rho_o U$ is dependent only upon the initial conditions and the constant has been freed to vary with λ. In addition, an examination of the available literature indicates that the density ratio to the half power has a significant effect on both the entrainment[5] and axis velocity decay[6] rates of variable density jet flows. The inclusion of this factor leads to the final version of the model,

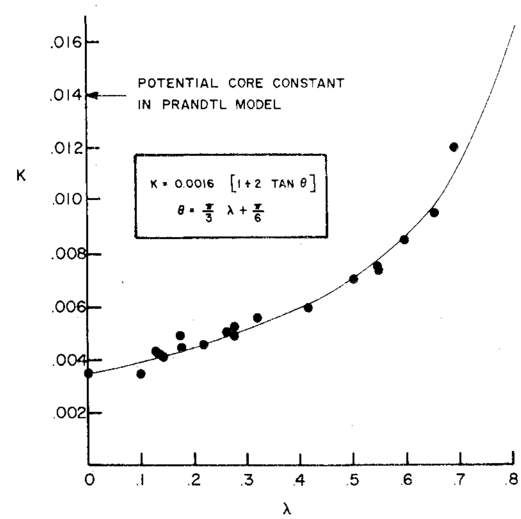

Fig. 1 Variation of potential core constant, K, with mass flux ratio, λ. ●: Empirical results, —: Curve fit.

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Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions; VTOL Aircraft Design.

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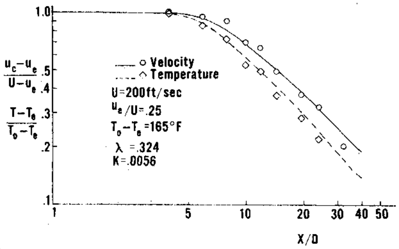

Fig. 2 Heated jet into coflowing stream, λ = 0.324, *K* = 0.0056, (from Landis and Shapiro,[9] Series B).

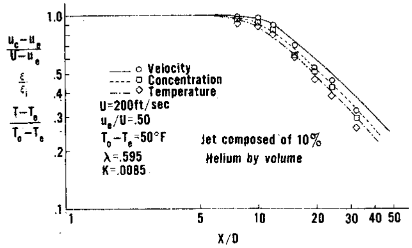

Fig. 3 Heated helium—air jet into coflowing stream, λ = 0.595, *K* = 0.0085, (from Landis and Shapiro,[9] Series L).

$$\rho\epsilon = K(\lambda)b(\rho_c u_c - \rho_e u_e)\left(\frac{\rho_e}{\rho_o}\right)^{1/2}\left[\frac{\rho_o U - \rho_e u_e}{\rho_c u_c - \rho_e u_e}\right]^{\lambda} \qquad (5)$$

It now remains to determine the variation of $K(\lambda)$ empirically and thereby illustrate the applicability of the model. In an effort to retain the simplicity of the model, the constant in the second region of the flow (i.e., u_c < 0.99U) is specified as equal to the second constant of the Prandtl model, $K = 0.022$. The constant in the first region of the flow is chosen empirically and depends upon the mass flux ratio λ. The choice of $K(\lambda)$ is essentially equivalent to specifying the length of the potential core as a function of mass flux ratio, λ. Thus the success of this model is due in large part to the empirical specification of decreased mixing within the potential core.

The empirical variation of K with mass flux ratio λ is shown in Fig. 1 along with a curve fit. The results were obtained by producing the best match of centerline decay of velocity, temperature, and concentration of axisymmetric jet experiments from ten different sources. The axisymmetric geometry was chosen because of the relative abundance of available experiments. Most of the available data involve the decay of axial velocity and this variable was therefore chosen for the comparison between predictions and experiments. The computations were carried out using a modification of a program originally due to Edelman and Fortune[7] and included the following areas: a) constant density flow; b) heated air jets into ambient air; c) heated air jets into coflowing air streams; d) variable density jets into coflowing streams. A Prandtl number of 0.72 was defined for the air into air cases while an approximate Prandtl number relationship for the nonuniform density cases was assumed from Fig. 16 of Ref. 8. The Lewis number was assumed unity.

Two computational results which illustrate the applicability of the simple model to thrust augmenting ejector flow conditions are shown in Figs. 2 and 3. Both are from experiments performed by Landis and Shapiro.[9] Figure 2 involves a substantial temperature difference between the primary and coflowing streams while Fig. 3 involves a difference in composition as well as temperature. Both of these situations are likely to arise in application and the predictions of the simple model may be seen to be quite reasonable.

References

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